Are There Low-Lying Intruder States in ^8Be?

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I. INTRODUCTION AND MOTIVATION

In an R matrix analysis of the $\beta^{\mp}$-delayed alpha spectra from the decay of ^8Li and ^8B as measured by Alburger, Donavan and Wilkinson [1], Warburton [2] made the following statement in the abstract: "It is found that satisfactory fits are obtained without introducing intruder states below 26-MeV excitations". However, Barker has questioned this [3,4] by looking at the systematics of intruder states in neighboring nuclei. He noted that the excitation energies of 0_2^+ states in ^{16}O, ^{12}C and ^{10}Be were respectively 6.05 MeV, 7.65 MeV and 6.18 MeV. Why should there not then be an intruder state in ^8Be around that energy?

In recent works [5,6] the current authors and S. S. Sharma allowed up to $2\hbar\omega$ excitations in ^8Be and in ^{10}Be, and indeed $2p-2h$ intruder states were studied with some care in ^{10}Be. Using a simple quadrupole-quadrupole interaction $-\chi Q\cdot Q$ with χ=0.3615 MeV/fm^4 for ^{10}Be and $\hbar\omega = 45/A^{1/3} - 25/A^{2/3}$. We found a $J=0^+$ intruder state at 9.7 MeV excitation energy. This is higher than the experimental value of 6.18 MeV, but it is in the ballpark. However, there are other $J=0^+$ excited states below the intruder state found in the calculation.

In a $0p$-shell calculation with the interaction $-\chi Q\cdot Q$, using a combination of the Wigner Supermultiplet theory [7] characterized by the quantum numbers $[f_1 f_2 f_3]$ and Elliott's $SU(3)$ formula [10], one can obtain the following expression giving the energies of the various states:

$$E(\lambda\ \mu) = \bar{\chi}\left[-4(\lambda^2+\mu^2+\lambda\mu+3(\lambda+\mu))+3L(L+1)\right]$$

where

$$\lambda = f_1 - f_2, \quad \mu = f_2 - f_3$$

and

$$\bar{\chi} = \chi\frac{5b^4}{32\pi} \qquad (b^2 = \frac{\hbar}{m\omega})$$

The two $J=0^+$ states lying below the calculated intruder state in ^{10}Be, at least in the calculation, correspond to two degenerate configurations [411] and [330]. Both of these have configurations $L=1$ $S=1$ from which one can form the triplet configurations $J=0^+,\ 1^+,\ 2^+$. Hence, besides the intruder state, we have the above two $J=0^+$ states as candidates for the experimental 0_2^+ state at 6.18 MeV.

As noted in the previous work [5] if, in the $0p$-shell model space we fit χ to get the energy of the lowest 2^+ state in ^{10}Be to be at the experimental value of 3.368 MeV ($18\bar{\chi}$), then the two sets of triplets are at an excitation energy of 30 $\bar{\chi}$ which equals 5.61 MeV -not far from the experimental value. There is however a problem -in a $0p$-space calculation with $Q\cdot Q$, the lowest 2^+ state is two-fold degenerate, corresponding to $J=2^+$ $K=0$ and $J=2^+$ $K=2$.

So it is by no means clear if the 0^+ state in ^{10}Be at 6.18 MeV is an intruder state. We will discuss this more in a later section. It should be noted that in the previously mentioned calculation [6], the energy of the intruder state is very sensitive to the value of χ, the strength of the $Q\cdot Q$ interaction. The energy of this intruder state drops down rapidly and nearly linearly with increasing χ.

II. RESULTS

In tables I, II and III we give results for the energies of $J=0^+$ and 2^+ states in ^8Be, in which up to $4\hbar\omega$ excitations are allowed relative to the basic configurations $(0s)^4(0p)^4$. The different tables correspond to different interactions as follows:

(a) Quadrupole-Quadrupole: $V=-\chi Q\cdot Q$ with $\chi=0.3467\ MeV/fm^4$.

(b) $V=-\chi Q\cdot Q\ +\ xV_{s.o.}$ (χ as above and $x=1$).

(c) $V=V_c+xV_{s.o.}+yV_t$ ($x=1,\ y=1$).

In the above, *s.o.* stands for spin-orbit, t for tensor and c for central. V is a *two-body* interaction which for $x=1,\ y=1$ gives a good fit to the Bonn A non-relativistic G matrix elements. This has been discussed extensively in previous references [5,9].

In tables IV, V and VI we present results for isospin one $J=0^+$ and 2^+ states in ^{10}Be in which up to $2\hbar\omega$ excitations have been included. We have the same three interactions as above but with $\chi=0.3615\ MeV/fm^4$ in (a) and (b).

In all the tables we give the excitation energies of the $J=0^+$ and 2^+ states and the percent probability that there are no excitations beyond the basic configuration ($0\hbar\omega$) and the percentage of $2\hbar\omega$ excitations (as well as $4\hbar\omega$ excitations for ^8Be).

Note that for interaction (a) the respective percentages for the ground state of ^8Be (see table I) are 62.8%, 25.7% and 11.5%: there is considerable mixing. Thus we should not forget, when we discuss the question "where are the intruder states?", that there is considerable admixing of $2\hbar\omega$ and $4\hbar\omega$ excitations *in the ground state*. Note that the ground state configuration does not change very much for the three interactions that are considered here. For example, as seen in table III, the corresponding percentages for the (x,y) interaction are 62.2%, 26.2% and 11.6%.

By looking at these tables, it is not too difficult to see at what energies the intruder states set in. One sees a sharp drop in the $0\hbar\omega$ occupancy. For example in table I, whereas the $0\hbar\omega$ percentage for the 18.7 MeV and 20.2 MeV states are respectively 93.9% and 94.6%, for the next state at 26.5 MeV the percentage drops to 29.4% -also the next four states listed have very low $0\hbar\omega$ percentages and are therefore intruders.

Let us somewhat arbitrarily define an intruder state as one for which the $0\hbar\omega$ percentage is less than 50%. By this criterion, and for the three interactions discussed here, the lowest $J=0^+$ intruder states in ^8Be are at 26.5 MeV, 26.5 MeV and 28.7 MeV (see tables I,II and III). The $J=2^+$ intruder states are at 27.5 MeV, 27.5 MeV and 33.7 MeV. Note that up to $4\hbar\omega$ excitations were allowed in these calculations. These energies are very high and would argue against the suggestion by Barker that there are low-lying intruder states in ^8Be.

What about ^{10}Be? Remember that in this nucleus we only include up to $2\hbar\omega$ excitations. For the three interactions considered, the lowest $J=0^+$ $T=1$ intruder states are at 9.7 MeV, 11.4 MeV and 31.0 MeV. The 'anomalous' behavior for the last value (31.5 MeV for the (x,y) interaction) will be discussed in a later section.

Note that when a spin-orbit is added to $Q\cdot Q$, the energy of the intruder state goes up *e.g.* 11.4 MeV *vs* 9.7 MeV. The lowest-lying $J=2^+$ $T=1$ intruder states are at 11.9 MeV, 13.8 MeV and 33.4 MeV. The energy of the non-intruder ($L=1$ $S=1$) $J=0^+,\ 1^+,\ 2^+$ triplet also goes up as can be seen from tables IV and V.

For the two $Q\cdot Q$ interactions, the energies of the intruder states in ^{10}Be are much lower than in ^8Be. This conclusion still holds if we were to use ^8Be energies calculated in $(0{+}2)\hbar\omega$ configuration space -see table VII. This would indicate that even if we do find low-lying intruder states in ^{10}Be, such a finding in itself is not proof that they are also present in ^8Be. Indeed, our calculations would dispute this claim.

III. $(0+2)\hbar\omega$ VS $(0+2+4)\hbar\omega$ CALCULATIONS FOR ^8BE

In table VII we show the results for the energy of the first intruder state in ^8Be in calculations in which only up to $2\hbar\omega$ excitations are included, and compare them with the corresponding results for up to $4\hbar\omega$. For interactions (a) and (b), the value of χ was changed to 0.4033 MeV/fm^4 in order that the energy of the 2_1^+ state come close to experiment. In more detail, we have to rescale χ depending on the model space in order to get the 2_1^+ state at the right energy. In general, the more $np-nh$ configurations we include the smaller χ is.

We see that in the larger-space calculation $(0+2+4)\hbar\omega$, the energies of the lowest intruder states in most cases come down about 5 MeV relative to the $(0+2)\hbar\omega$ calculation. The excitation energies are still quite high, however, all being above 25 MeV. One possible reason for the difference between the results of the two calculations is that in the $(0+2)\hbar\omega$ calculation there is level repulsion between the $0\hbar\omega$ and the $2\hbar\omega$ configurations, and that the $4\hbar\omega$ configurations are needed to repel the $2\hbar\omega$ states back down.

IV. THE FIRST EXCITED $J=0^+$ STATE OF ^{10}BE

Is the first excited $J=0^+$ state in ^{10}Be an intruder state or is it dominantly of the $(0s)^4(0p)^6$ configuration? Experimentally, very few states have been identified in ^{10}Be. The known positive-parity states are as follows [11]:

J^π	$E_x(MeV)$
0_1^+	0.000
2_1^+	3.368
2_2^+	5.959
0^+	6.179
2^+	7.542
(2^+)	9.400

In the $(0s)^4(0p)^6$ calculation with a $Q{\cdot}Q$ interaction, the lowest 2^+ state at $18\bar{\chi}$ is doubly degenerate and corresponds to $K=0$ and $K=2$ members of the [42] configuration. There are two degenerate ($L=1$ $S=1$) configurations at $30\bar{\chi}$ with supermultiplet configurations [330] and [411]. From $L=1$ $S=1$ one can form a triplet of states with $J=0^+,\ 1^+,\ 2^+$. If we choose $\bar{\chi}$ by getting the 2_1^+ state correct at 3.368 MeV, then the two $L=1$ $S=1$ triplets would be at $30/18\times 3.36$ $MeV=5.61$ MeV. However, there should be a *triplet* of states. In more detailed calculations, as the spin-orbit interaction is added to the $Q\cdot Q$ interaction, the triplet degeneracy gets removed with the ordering $E_{2^+}<E_{1^+}<E_{0^+}$. As seen in tabel IV, the $J=0^+$ and 2^+ states of ^{10}Be at 3.7 MeV and 7.3 MeV are degenerate with a pure $Q\cdot Q$ interaction. This is also true for $J=1^+$. In table V, however, when the spin-orbit interaction is added to $Q\cdot Q$, we find that whereas the 0_2^+ is at 8.0 MeV, the 2_3^+ state is at 6.8 MeV.

Hence if the 0^+ state at 6.179 MeV were dominantly an $L=1$ $S=1$ non-intruder state, one would expect a $J=1^+$ and a $J=2^+$ state at lower energies. Thus far no $J=1^+$ level has been seen in ^{10}Be but this is undoubtedly due to the lack of experimental research on this target. Now there *is* a lower 2^+ state at 5.959 MeV. This could be a member of the $L=1$ $S=1$ triplet *or* it could be the $K=2$ state of the [42] configuration.

Hence, one possible scenario is that indeed the 2_2^+ state is dominantly of the [42] configuration and the $J=0_2^+$ state is a singlet. This would support the idea that the $J=0_2^+$ state is an intruder state. The second scenario has the $J=2_2^+$ state being dominantly an $L=1$ $S=1$ state for which the $J=1^+$ member has somehow not been found. This would be in support of the idea that the 0_2^+ state is *not* an intruder state.

Let us look in detail at tables IV, V and VI which show where the energies of the intruder states are in a $(0{+}2)\hbar\omega$ calculation. For the $Q \cdot Q$ interaction (with $\chi = 0.3615\ MeV/fm^4$), the lowest $J = 0^+$ intruder state is at 9.7 MeV and the lowest $J = 2^+$ intruder state is at 11.9 MeV. These energies are *much lower* than the corresponding intruder state energies for ^8Be. This in itself is enough to tell us that the presence of a low-energy intruder state in ^{10}Be does not imply that there should be a low energy intruder state in ^8Be. Note that the intruder states in this model space and with this interaction have 100% '$2\hbar\omega$' configurations. This has been noted and discussed in [6] and is due to the fact that the $Q \cdot Q$ interaction cannot excite two nucleons from the N shell to the $N \pm 1$ shell.

Still, in table IV, there are two $J = 0^+$ states (below the intruder state) at 3.7 MeV and 7.3 MeV. Even in this large-space calculation, these are members of degenerate $L = 1\ S = 1$ triplets $J = 0^+,\ 1^+,\ 2^+$. Indeed, if we look down the table, we see the 3.7 MeV and 7.3 MeV values in the $J = 2^+$ column.

In table V, when we add the spin-orbit interaction to $Q \cdot Q$, the energies of the 0_2^+ and 0_3^+ states go up, but so does the energy of the $J = 0_4^+$ intruder state. The energies of the 0_2^+, 0_3^+ and 0_4^+ (intruder) states in table IV are 3.7, 7.3 and 9.7 MeV; in table V, with the added spin-orbit interaction they are 8.0, 9.6 and 11.4 MeV.

In table VI we show results of an up-to-$2\hbar\omega$ calculation with the realistic interaction. Here, we see a drastically different behavior for the intruder state energy in ^{10}Be. The lowest $J = 0^+$ intruder state is at 31.0 MeV, and the lowest $J = 2^+$ intruder state is at 33.4 MeV (recall our operational definition -an intruder state has less than 50% of the $0\hbar\omega$ configuration). For the $Q \cdot Q$ interaction, in contrast, the intruder state was at a much lower energy. A possible explanation is that for the (x, y) interaction, unlike $Q \cdot Q$, one *does have* large off-diagonal matrix elements in which two nucleons are excited from N to $N \pm 1$ *e.g.* from $0p$ to $1s - 0d$. This will cause a large level repulsion between the $0\hbar\omega$ and the $2\hbar\omega$ configurations and drive them far apart. Presumably, if we included $4\hbar\omega$ configurations into the model space, they would push the $2\hbar\omega$ configurations back down to near their unperturbed positions.

Thus, the problem is rather difficult to sort out theoretically, so we can at best suggest that more experiments be done on ^{10}Be. For example, the $B(E2)$ to the 2_2^+ state would be useful. There should be a much larger B(E2) to the $L = 2\ K = 2$ member of a [42] configuration than to an $(L = 1\ S = 1)$ state. We also predict a substantial $B(M1) \uparrow$ to the first $J = 1^+\ T = 1$ state in ^{10}Be. Whereas with a pure $Q \cdot Q$ interaction the $B(M1)$ to this state would be zero, the presence of a spin-orbit interaction will 'light up' the 1_1^+ state in ^{10}Be. The $J = 1^+$ should be seen.

V. CONCLUSIONS

Because of the important implications to astrophysics of the ^8Be nucleus, we feel that Barker's suggestion of looking for intruder states in this and neighboring nuclei is well founded. In all our calculations, the positive-parity intruder states in ^8Be come at a very high excitation energy -greater than 26 MeV, thus supporting the statement by E. Warburton in the abstract of his 1986 work [2]. However, because of significant differences between the realistic and the $Q \cdot Q$ interactions for the predicted energies of intruder states in ^{10}Be, and because of the possibility of low-lying non-intruder states *e.g.* $L = 1\ S = 1$ triplets, we cannot determine with certainty whether or not the first excited state in ^{10}Be is an intruder state. However, even if it is, this does not mean that there has to be a low-energy intruder state in ^8Be. Indeed, our $Q \cdot Q$ calculations clearly contradict this claim, and in ^8Be, our realistic-interaction calculations lead to the same conclusion.

VI. ACKNOWLEDGEMENTS

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TABLE I. $J = 0^+$ and 2^+ states in ^8Be for the interaction $-\chi Q \cdot Q$ with $\chi = 0.3457\ MeV/fm^4$ with up to $4\hbar\omega$ excitations allowed. The percentage of $0\hbar\omega$, $2\hbar\omega$ and $4\hbar\omega$ occupancies are given, as well as the $B(E2)(0_1^+ \rightarrow 2_i^+$.

		(a) $J = 0^+$ $T = 0$ States		
$E_{exc}(MeV)$	$0\ \hbar\omega$	$2\ \hbar\omega$	$4\ \hbar\omega$	
0.0	62.8	25.7	11.5	
11.9	82.2	11.6	6.2	
16.6	94.0	2.2	3.8	
18.7	93.9	2.7	3.4	
20.3	94.6	2.3	3.2	
26.5	29.4	49.7	20.9	
29.9	4.1	75.8	20.1	
32.4	0.0	85.4	14.6	
34.5	0.0	86.2	13.9	
36.4	14.8	69.3	15.8	
		(b) $J = 2^+$ $T = 0$ States		
$E_{exc}(MeV)$	$0\ \hbar\omega$	$2\ \hbar\omega$	$4\ \hbar\omega$	$B(E2)_{0_1^+ \rightarrow 2_i^+}$ $(e^2 fm^4)$
3.1	64.3	25.0	10.7	67.2
11.9	82.2	11.6	6.2	0.0
14.2	85.1	9.6	5.3	0.0
16.7	87.6	8.0	4.4	0.0
16.7	92.9	3.2	3.9	0.0
18.7	93.9	2.7	3.4	0.0
18.7	93.9	2.7	3.4	0.0
20.3	94.6	2.3	3.2	0.0
27.5	29.9	49.6	20.5	15.2
30.4	0.0	78.4	21.6	0.0
32.0	1.2	79.1	19.7	1.7
32.4	0.0	85.4	14.6	0.0
34.2	0.1	82.5	17.4	0.0
34.5	0.0	86.1	13.9	0.0
36.2	11.6	73.7	14.7	0.0

TABLE II. Same as Table I but for the interaction $-\chi Q \cdot Q + xV_{s.o.}$ with $\chi = 0.3457\ MeV/fm^4$ and $x = 1$.

(a) $J = 0^+$ $T = 0$ States

$E_{exc}(MeV)$	0 $\hbar\omega$	2 $\hbar\omega$	4 $\hbar\omega$
0.0	65.1	24.0	10.9
12.8	83.6	10.3	6.1
16.4	89.7	6.0	4.3
21.9	91.7	4.6	3.7
26.4	69.3	21.3	9.4
26.5	40.7	44.0	15.3
29.9	3.4	77.4	19.2
32.1	0.0	86.6	13.4
37.3	0.0	85.6	14.3
38.4	18.2	66.2	15.6

(b) $J = 2^+$ $T = 0$ States

$E_{exc}(MeV)$	0 $\hbar\omega$	2 $\hbar\omega$	4 $\hbar\omega$	$B(E2)_{0_1^+ \to 2_i^+}$ $(e^2 fm^4)$
3.1	66.7	23.3	10.1	63.4
10.2	85.8	8.8	5.4	0.4
13.2	88.2	7.2	4.6	0.9
16.2	91.9	4.2	3.9	0.0
17.7	86.4	8.9	4.7	0.2
19.6	88.3	7.4	4.3	0.0
21.6	84.8	10.3	4.9	0.1
22.2	91.0	5.1	3.8	0.0
27.5	27.8	53.1	19.1	14.5
30.9	0.9	78.0	21.0	0.0
31.9	1.1	80.2	18.7	1.6
32.4	0.0	86.2	13.8	0.0
34.3	0.2	85.7	14.0	0.0
34.6	1.2	83.8	15.1	0.1
35.2	11.4	74.0	14.6	0.1

TABLE III. Same as Table I but for the realistic (x, y) interaction with $x = 1$ and $y = 1$.

(a) $J = 0^+$ $T = 0$ States

$E_{exc}(MeV)$	$0\ \hbar\omega$	$2\ \hbar\omega$	$4\ \hbar\omega$
0.0	62.2	26.2	11.6
22.8	66.5	23.6	9.9
28.7	6.5	71.0	22.5
30.3	66.5	23.0	10.5
35.3	67.5	22.4	10.1
39.4	7.3	73.4	19.3
43.5	56.3	31.4	12.3
47.6	8.8	70.5	20.7
49.5	2.3	76.7	21.6
50.1	3.3	75.7	21.0

(b) $J = 2^+$ $T = 0$ States

$E_{exc}(MeV)$	$0\ \hbar\omega$	$2\ \hbar\omega$	$4\ \hbar\omega$	$B(E2)_{0_1^+ \rightarrow 2_i^+}$ $(e^2 fm^4)$
5.4	62.2	26.6	11.1	31.1
19.5	70.0	20.4	9.6	0.0
21.5	69.5	20.2	10.3	0.1
26.2	69.7	20.5	9.8	0.4
30.4	70.2	20.9	8.9	0.0
31.0	56.7	30.9	12.6	1.7
33.7	13.5	65.7	20.8	3.7
35.1	71.3	19.7	9.0	0.0
38.2	67.7	22.4	9.8	0.0
41.6	9.0	68.8	22.2	1.3
45.0	1.0	79.7	19.3	0.1
45.9	2.9	77.9	19.2	2.4
46.3	3.2	76.7	20.1	1.3
47.3	0.3	79.5	20.2	0.0
48.4	1.5	79.8	18.6	0.0

TABLE IV. $J = 0^+$ and 2^+ states in ^{10}Be for the interaction $-\chi Q \cdot Q$ with $\chi = 0.3615\ MeV/fm^4$ with up to $2\hbar\omega$ excitations allowed. The percentage of $0\hbar\omega$ and $2\hbar\omega$ occupancies are given, as well as the $B(E2)(0_1^+ \rightarrow 2_i^+$.

(a) $J = 0^+$ $T = 1$ States			
$E_{exc}(MeV)$	0 $\hbar\omega$	2 $\hbar\omega$	
0.0	81.8	18.2	
3.7	81.0	19.0	
7.3	93.6	6.4	
9.7	0.0	100.0	
12.1	92.9	7.1	
12.1	92.9	7.1	
13.9	93.1	6.9	
17.7	98.9	1.1	
22.1	0.0	100.0	
22.9	0.0	100.0	
(b) $J = 2^+$ $T = 1$ States			
$E_{exc}(MeV)$	0 $\hbar\omega$	2 $\hbar\omega$	$B(E2)_{0_1^+ \rightarrow 2_i^+}$ $(e^2 fm^4)$
2.2	81.3	18.7	5.0
3.4	83.4	16.6	47.2
3.7	81.0	19.0	0.0
7.3	93.6	6.4	0.0
9.2	82.9	17.1	0.0
10.9	91.9	8.1	0.0
11.9	0.0	100.0	0.0
12.1	92.9	7.1	0.0
12.1	92.9	7.1	0.0
12.1	92.9	7.1	0.0
13.9	93.1	6.9	0.2
13.9	93.1	6.9	0.0
13.9	93.1	6.9	0.0
17.7	98.9	1.1	0.0
22.1	0.0	100.0	0.0

TABLE V. Same as Table IV but for the interaction $-\chi Q \cdot Q + xV_{s.o.}$ with $\chi = 0.3615\ MeV/fm^4$ and $x = 1$.

(a) $J = 0^+$ $T = 1$ States

$E_{exc}(MeV)$	0 $\hbar\omega$	2 $\hbar\omega$
0.0	85.6	14.4
8.0	80.8	19.2
9.6	92.0	8.0
11.4	0.0	100.0
12.1	91.5	8.5
16.4	90.6	9.4
19.7	90.5	9.5
23.1	88.7	11.3
24.0	0.0	100.0
26.1	0.0	100.0

(b) $J = 2^+$ $T = 1$ States

$E_{exc}(MeV)$	0 $\hbar\omega$	2 $\hbar\omega$	$B(E2)_{0_1^+ \to 2_i^+}$ $(e^2 fm^4)$
3.0	85.5	14.5	40.1
4.6	83.7	16.3	3.4
6.8	90.8	9.2	0.3
7.8	83.5	16.5	3.7
11.8	84.8	15.2	0.1
13.0	91.2	8.8	0.1
13.8	0.0	100.0	0.0
14.1	90.9	9.1	0.0
14.8	90.9	9.1	0.0
15.5	90.3	9.7	0.0
17.2	90.0	10.0	0.1
17.2	88.0	12.0	0.0
18.2	90.3	9.7	0.1
21.2	89.0	11.0	0.0
23.0	52.8	47.3	0.0

TABLE VI. Same as Table IV but for the realistic (x, y) interaction with $x = 1$ and $y = 1$.

(a) $J = 0^+$ $T = 1$ States		
$E_{exc}(MeV)$	0 $\hbar\omega$	2 $\hbar\omega$
0.0	73.3	26.7
8.7	74.4	25.6
12.0	74.7	25.3
21.1	76.5	23.5
23.7	77.5	22.5
31.0	49.3	50.7
31.5	25.4	74.6
34.5	5.8	94.2
37.6	0.6	99.4
39.7	74.1	25.9

(b) $J = 2^+$ $T = 1$ States			
$E_{exc}(MeV)$	0 $\hbar\omega$	2 $\hbar\omega$	$B(E2)_{0_1^+ \to 2_i^+}$ $(e^2 fm^4)$
4.6	73.5	26.5	19.7
5.2	73.9	26.1	3.2
9.2	73.7	26.3	1.5
10.1	75.8	24.2	0.0
17.4	74.5	25.5	0.0
19.7	75.7	24.3	0.1
20.2	77.0	23.0	0.0
22.1	76.9	23.1	0.2
22.9	77.1	22.9	0.0
23.7	77.2	22.8	0.0
27.2	76.8	23.2	0.0
29.0	76.9	23.1	0.2
32.5	76.9	23.1	0.0
33.4	0.3	99.7	0.0
35.5	71.7	28.3	0.2

TABLE VII. Excitation energies (in MeV) of the first $J = 0^+$ and 2^+ intruder states in ^8Be: a comparison of up to $2\hbar\omega$ and up to $4\hbar\omega$ calculations for the three interactions.

	$Q \cdot Q$	$Q \cdot Q + xV_{s.o.}$	(x, y)=(1,1)
		$J = 0^+$ $T = 0$ States	
$2\hbar\omega$	32.1	30.1	33.8
$4\hbar\omega$	26.5	26.5	28.7
		$J = 2^+$ $T = 0$ States	
$2\hbar\omega$	31.5	30.9	36.6
$4\hbar\omega$	27.5	27.5	33.7